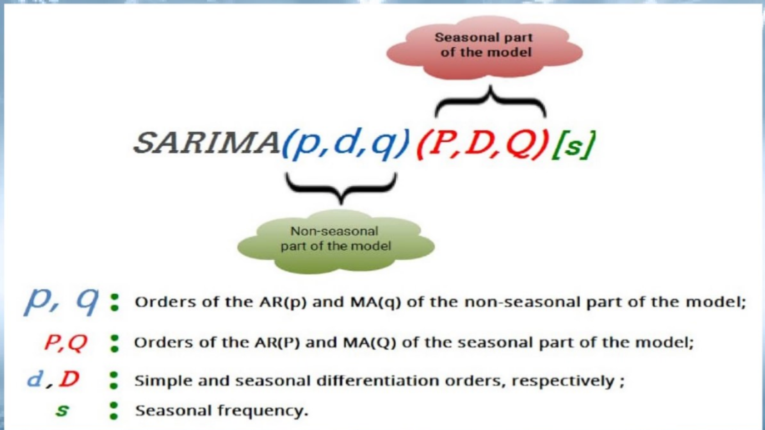
SARIMA models

The use of SARIMA models constitutes an important aspect of statistical applications in economics and finance. These applications began with Boxet Jenkins (1976). In order to make the methodology more flexible and more accessible to a larger number of users, many expert systems have emerged.


Seasonal part of the model
SARIMA(p,d,q) (P,D,Q) [s]
Non-seasonal part of the model
p, q : Orders of the AR(p) and MA(q) of the non-seasonal part of the model;
P,Q : Orders of the AR(P) and MA(Q) of the seasonal part of the model;
d, D : Simple and seasonal differentiation orders, respectively ;
s : Seasonal frequency.

definition

Lets p,q,d and $0 \leq s$ and , a (X_t) is a SARIMA $(p, d, q)(P, D, Q)_s$ process if

$$Y_t = (1 - B)^d (1 - B^s)^D X_t$$

is a stationary ARMA process of the form

$$\Phi(B)F(B^s)Y_t = \Theta(B)G(B^s)\eta_t$$

$\Phi(resp.\Theta) is\ the\ pol.generator 1 AR(p)(resp.MA(q))$:
$\Phi(Z) = 1 - \sum_{k=1}^{p} \phi_k Z^K$ and $\Theta(Z) = 1 - \sum_{k=1}^{q} \theta_k Z^K$
and where, for the seasonality $Y_t - Y_{t-s}$,
F (resp. G) is the pol. generator of an AR(P) (resp. MA(Q)):
$F(Z) = 1 - \sum_{k=1}^{p} f_k Z^K$ *and* G(Z)=1-$\sum_{k=1}^{q} g_k Z^K$

- Y_t is a particular ARMA process $(p + Ps, q + Qs)$ is stationary.

examples

- Let

$$X_t = \mu + S_t + \eta_t$$

with $S_t = S_{t+s}$ t>0 is process SARIMA(0,0,0)(0,1,1)$_s$

$$Y_t = (I - B^s)X_t = \eta_t - \eta_{t-s}$$

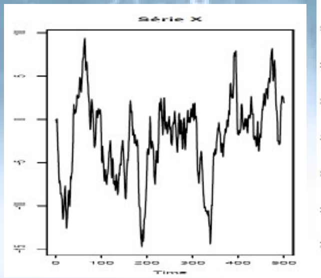
is ARMA(0,s) process(non inversible)

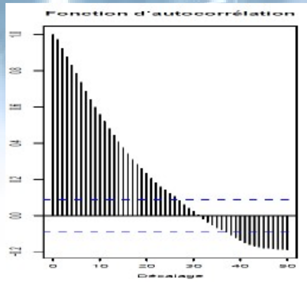
- if we study SARIMA(1,1,0)(1,1,0)$_{12}$model we will find $s = 12$
$p = P + 1, q = 0 = Q, d = 1 = D$
$so\ Y_t will\ be : Yt = (I - B)(I - B^{12})X_t and\ we\ this\ relation$

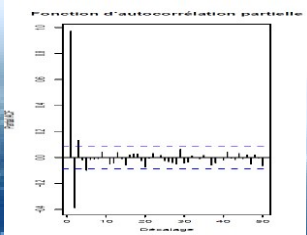
$$Y_t = \phi Y_{t-1} + fY_{t-12} - \phi f Y_{t-13} + \eta_t$$

because $\Phi(z)F(z^s)Y_t = (1 - \phi z)(1 - fz^{12})$

- Let SARIMA trajectory, correlogigram and partial correlogigram of the process SARIMA $(1,1,1)(0,0,0)_{12}$







Forecasting

If the data $X_1, \ldots, X_T$ follow SARIMA$(p, d, q)(P, D, Q)_s$ defined

$$Y_t = (1-B)^d(1-B^s)^D X_t$$

for example $(d, D, s) = (1, 1, 12)$

$$Y_t = (X_t - X_{t-12}) - (X_{t-1} - X_{t-13})......(1)$$

We treat the problem of predicting the process Y_t of the type $ARMA(p + P_s, q + Q_s)$ as seen previously and we return after at the level of the X_t process of the SARIMA type by expressing $X_T(1)$ as a function of $Y_T(1)$ and the observed values X_t for $T \leq t$.

example

$Y_t = \phi Y_{t-1} + fY_{t-12} - f\phi Y_{t-13} + \epsilon_t$
$\Rightarrow \hat{y}_{T+1} = \phi Y_T + fY_{T-11} - f\phi Y_{T-12}$
$\Rightarrow \hat{y}_{T+2} = \phi \hat{y}_{T+1} + fY_{T-10} - f\phi Y_{T-11}$
so the equation (1) implied preduction of a process
$X_t of\ type\ SARIMA(1,1,0)(1,1,0)_{12}$

$$\hat{X}_{T+1} = \hat{y}_{T+1} + X_{T-11} + X_T - X_{T-12}$$